8th World Congress on Structural and Multidisciplinary Optimization
June 1 - 5, 2009, Lisbon, Portugal

Metamodels for Optimisation of Post-buckling Responses in Full-scale Composite Structures

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1. Abstract
Extensive application of advanced composite materials such as Carbon Fibre Reinforced Plastics (CFRP) emerges in design of aerospace structural components. Outstanding weight-related stiffness and strength properties in combination with structural topology solutions may lead to exploitation of full-load bearing potential of composite structures as utilization of the post-buckling region. In order to fully exploit the load-carrying capacity of such structures an accurate and reliable simulation is indispensable. That, however, requires fast tools which are capable of simulating the structural behaviour beyond skin buckling bifurcation points deep into the post-buckling phenomena up to the collapse of structure. In this paper a metamodeling methodology is proposed for post-buckling simulation of cylindrical-stiffened fuselage structures. Proposed methodology for elaboration of the fast simulation procedure for axially loaded stiffened cylinder structures is based on utilization of space-filling design of experiments and parametric and non-parametric approximations. For determination of the most suitable metamodeling technique different methods are compared – second-order global polynomials, second-order Locally-Weighted Polynomials, adaptively constructed sparse polynomials, Radial Basis Functions, Kriging, Multivariate Adaptive Regression Splines, and Support Vector Regression. Continuous design variables (the structural geometrical dimensions) are used together with a discrete variable (number of stiffeners), thus allowing to scale the full-scale structure towards the stiffened panel designs. The proposed and validated simulation procedure is an efficient optimum design tool in elaboration of the trade of design and in assessment of parametrical sensitivity analysis. It enables elaboration of Pareto-optimal fronts which can be used in the optimum design guidelines to realise the full potential of the stiffened composite structures subjected to uniform axial compression.

2. Keywords: Metamodeling, post-buckling, stiffened composite structures, Pareto-optimality.

3. Introduction
Demanding requirements for industrial applications of carbon-epoxy-reinforced composite structures can be met by reducing their structural weight within the safe, however not yet certified, design boundaries. In particular, great potential exists for the future increase of effectiveness of stiffened composite structures by allowing post-buckling of the structural elements to occur during the exploitation of the structure [1,2]. Nevertheless, even with the dramatic increase of computation power within the last decade, current numerical procedures still are incompatible for the direct optimisation of the post-buckling behaviour of stiffened cylindrical composite structures with sufficient reliability and efficiency [3].
In the current research, a metamodeling methodology has been developed and validated for the design and optimisation of a cylindrical-stiffened fuselage structures, loaded in compression well beyond the initial buckling load. A stiffened fuselage structure is assumed to consist of several stiffened panels which are more widely experimentally tested and numerically verified in the literature [4,5,6]. By evaluating the interaction ratio between the full-scale structure and stiffened panels it is possible to decrease the design time while the design reliability should remain close to the original level. The methodology used to determine the post-buckling response behaviour of stiffened panels and structures mainly relies on applying simplifying assumptions using semi-empirical/empirical data [3]. By employing the finite element method and explicit analysis procedures, it is possible to simulate the post-buckling behaviour of stiffened panels without having to place the same emphases on simplifying assumptions or empirical data. Moreover it could be validated that both the curved panel and the full-scale cylinder designs have the same buckling and post-buckling mode shape, thus the post-buckling pattern control could be applied as additional response to increase the design reliability. Therefore the resulting design procedure provides a time/reliability effective analysis tool for the safe exploitation of composite full-scale stiffened structures under the axial compression.

4. Fast Design Procedure

Design of computer experiments and approximation models are essential for efficiency and effectiveness in engineering numerical analyses of complex systems in which designers have to deal with multi-disciplinary and multi-objective analysis using very complicated and expensive-to-run computer analysis codes. To cut down the computational cost, metamodels, also referred to as surrogate models, are constructed while treating the analysis codes as black boxes. Metamodels approximate the behaviour of the analysis codes as closely as possible while being computationally cheaper to evaluate [7,8,9]. The process of design optimisation involving metamodeling usually comprises three major steps which may be interleaved iteratively: 1) sample selection (known as design of experiments) [10,11,12]; 2) construction of the metamodel that best describes the behaviour of the problem and estimation of its predictive performance; 3) employment of the metamodel in the optimisation, design space exploration, what-if analysis and other tasks. Figure 1 depicts a metamodeling flowchart for the design of stiffened composite structures where sampling within the domain of interest is performed extracting the numerical pre-buckling and post-buckling responses and elaborating the numerical values by simplifying the load-shortening reactions. Theses response values are then approximated by means of parametric or non-parametric approximation functions. The developed metamodels can be further used for the design optimisation, weight savings, parametric sensitivity analysis, Pareto-optimality evaluations etc.

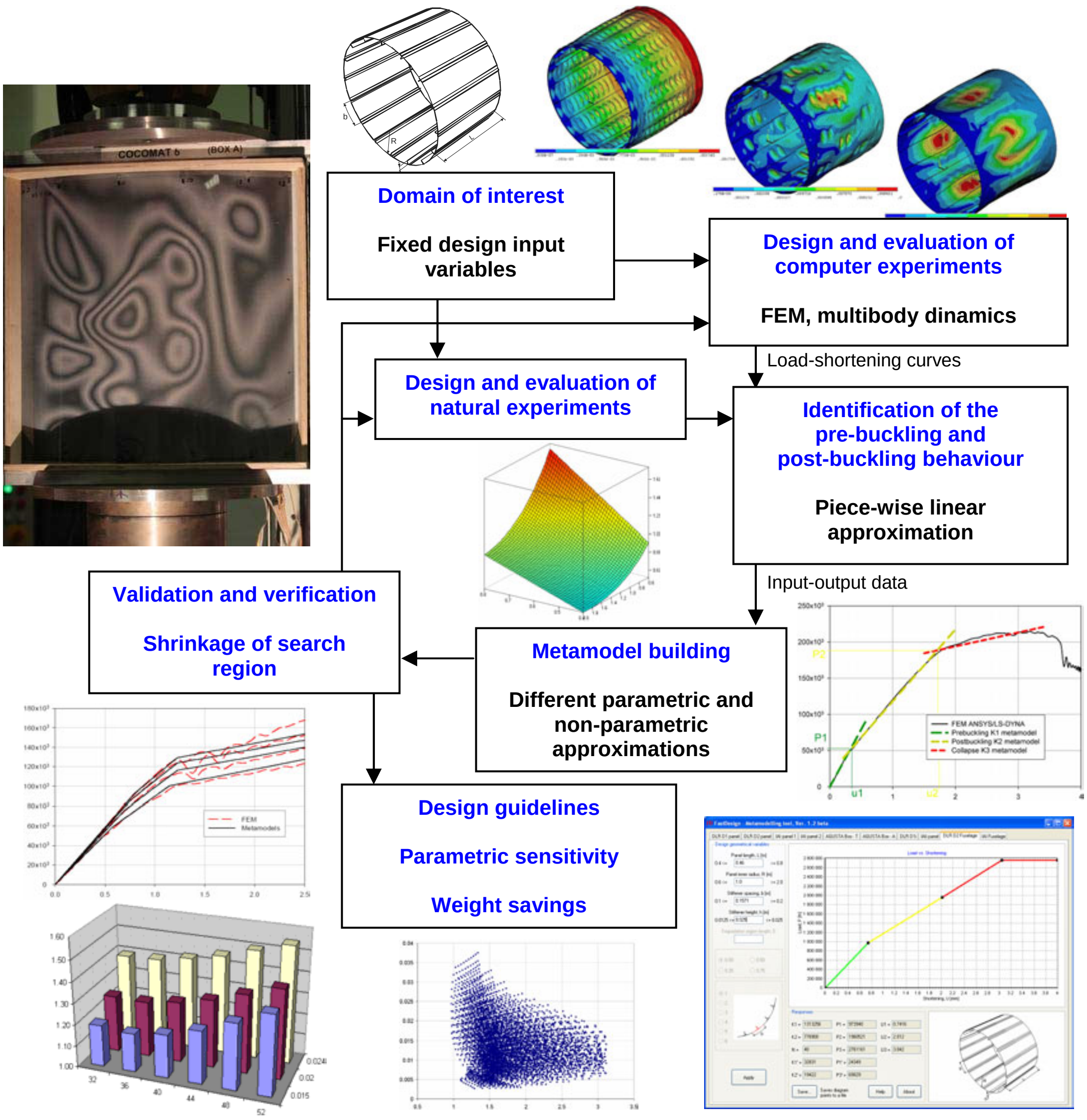

Figure 1: Metamodeling flowchart for design of full-scale stiffened composite structures

5. Metamodeling of Pre-buckling and Post-buckling Responses

The applied procedure is based on building of metamodels employing sequential experimental design [12] and both parametrical and non-parametrical approximation functions. The metamodels are built using stiffened fuselage structure geometrical variables extracting buckling/post-buckling structural responses. The numerical load-shortening responses, obtained from explicit FEM simulations by ANSYS/LS-DYNA (Figure 2) of composite stiffened structures subjected to buckling and post-buckling, have been simplified and the numerical values are extracted for the building of the metamodels.

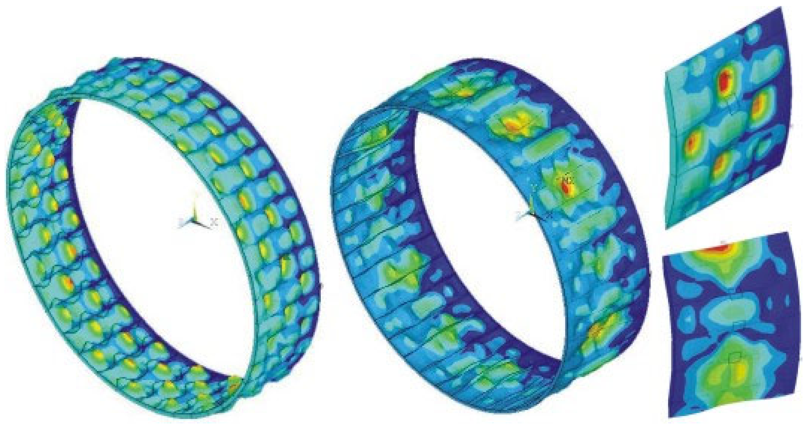

Figure 2: Typical post-buckling mode shape for full-scale stiffened structure and corresponding panel designs obtained with explicit ANSYS/LS-DYNA

5.1 Simplification Strategy for Load-shortening Response

It may be generalized that simplification of the load-displacement response in order to develop corresponding metamodels is based on the numerically obtained load-shortening curves (Figure 3), where the axial load *P*, stiffness *k*, and axial shortening *u* are functions of the design parameters. For this reason, the load-shortening curve is divided into three linear sections [9] representing pre-buckling load shortening, post-buckling load shortening, and the collapse region. Each section occupies a region where the load-shortening interconnection is linear and reaches the diverging point between the two linear curves, which is close to the skin buckling and stiffener buckling load obtained experimentally. Minimizing the discrepancy criterion allows controlling the diverging points between two correlated regions in the load-shortening curves. In validation by natural experiments, the numerical post-buckling critical load is more conservative than that obtained in physical tests [4,5,6]. Typical load-shorting of stiffened structure undergoing buckling and post-buckling response are shown in Figure 3 where corresponding simplifications have been overlaid indicating the k_1 pre-buckling, k_2 post-buckling, k_3 collapse regions stiffnesses as well as skin buckling P_1 and stiffener buckling load P_2.

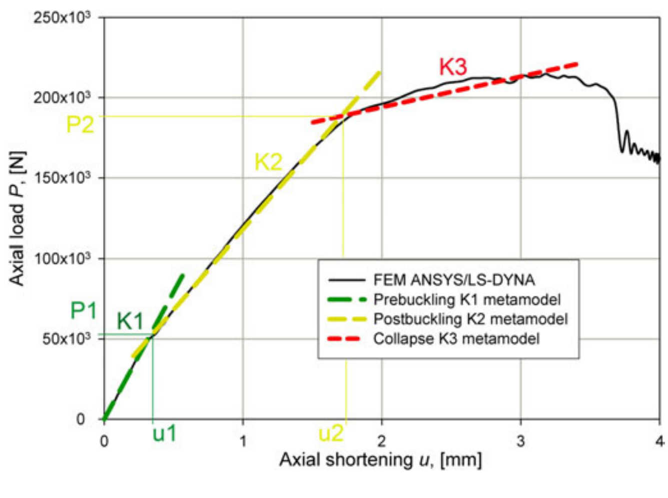

Figure 3: Typical load-shortening curve obtained with ANSYS/LS-DYNA overlaid with simplification approach subdividing load-shortening curve into three linear sections

5.2 Design Variables and FEM Model

Four-stiffener (Design 1) and five-stiffener (Design 2) panels with a regular distribution of the stiffeners around the arch (Figure 4), in order to represent the same post-buckling mode shape within the whole domain of interest, have been incorporated into the design of the full scale fuselage structure. Geometrical variables are taken as design configurations representing the particular domain of interest, where *L* is the panel length, *R* is the panel inner radius, *b* is the distance between the stiffeners, and *h* is the stiffener height (see Table 1).

Table 1: Stiffened panel structure with design geometrical variables

Name	Notation	Lower bound	Upper bound	Units
Panel length	*L*	400	800	mm
Panel inner radius	*R*	600	2000	mm
Stiffener spacing	*b*	100	200	mm
Stiffener height	*h*	12.5	30	mm

CFRP IM7/8552 laminate material with the following mechanical characteristics (fixed design parameters) was used for skin and stiffeners: Ex = 147.3 GPa; $Ey = Ez$ = 11.8 GPa; $Gxy = Gxz = Gyz$ = 6.0 GPa; ν = 0.3; ρ = 1600 kg/m³. The total thickness of the skin is s = 1 mm and the total thickness of the stiffener is *t* = 3 mm. For the skin, symmetric laminates with fixed ply angles $[90/\pm45/0]_s$ are considered similar to the symmetric laminate lay-up ply angles $[\pm45/0_2]_{3s}$ for the stringer. The stiffener was bonded to the skin using a one-step flange of 40 mm for the Design 1 and 48 mm for the Design 2. Clamped upper and lower edges and simply supported longitudinal edges were taken as panel boundary conditions [5,6].

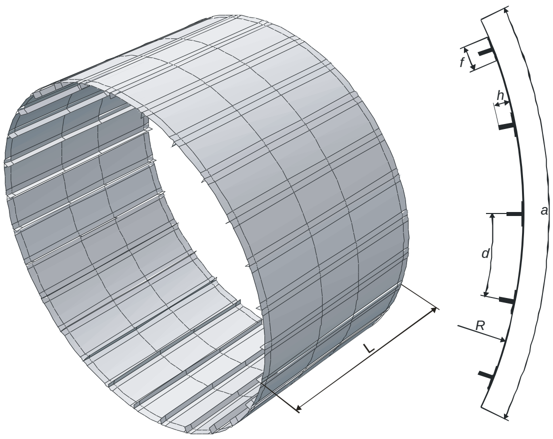

Figure 4: Stiffened structure with design geometrical variables

5.3 Metamodeling

A set of sequential design of computer experiments using the MSE space-filling criterion and the sequential point arranging method [12] were conducted for a four-variable design space with 51 sample points [9]. For accurate approximation of the responses seven approximation techniques were evaluated: full global second-order polynomials (FP), second-order Locally-Weighted Polynomials (LWP) [13,8], Radial Basis Function (RBF) interpolation [14], Kriging [15], Multivariate Adaptive Regression Splines (MARS) [16], Support Vector Regression (LVR) [17,18], and Adaptive Basis Function Construction (ABFC) [19,20,8]. LWP used the Gaussian weight function with the value of the bandwidth parameter found by leave-one-out cross-validation. RBF used the multi-quadric basis functions with the shape parameter equal to one. Kriging used first-order polynomial as a trend function and employed the Gaussian correlation function. MARS was that of piecewise-cubic type without special limitation of the number of basis functions. SVR used the Radial Basis Function kernel and the improved Sequential Minimal Optimisation algorithm [18] for which the complexity parameter and the gamma parameter were found using grid search and cross-validation from the range of values $\{10^{-1}, 10^{0}, 10^{1}, 10^{2}\}$ for the complexity parameter and $\{10^{-2}, 10^{-1}, 10^{0}, 10^{1}\}$ for the gamma parameter. ABFC involved the ensembling of the individually built sparse polynomials [20]. All the methods except SVR are implemented in the freely-available software tool VariReg [21] (in this study all the other less important settings not mentioned here were left at the default values). For SVR the implementation in the Weka software [22] was employed. Note that the employed source code for the Kriging technique was developed by [23].

To evaluate predictive performances of the built metamodels, in this study a 10-fold cross-validation method is used in which the full data set is divided in 10 equally (or approximately equally) sized subsets. In each of the 10 cross-validation iterations nine of the subsets are used for model building and one left subset is used as an independent test data set for evaluation of the built model. As the model accuracy measure the following average relative measure was used:

$$CVErr = 100\% \frac{1}{10} \sum_{j=1}^{10} \left(\frac{\sqrt{\frac{1}{n_j} \sum_{i=1}^{n_j} (y_{ji} - F_{-j}(x_{ji}))^2}}{\frac{1}{n_j} \sum_{i=1}^{n} y_{ji}} \right) \tag{1}$$

where y_{ji} is the real response value for the ith test point of the jth test set; $F_{-j}(x_{ji})$ is the predicted response value at the ith test point of the jth test set by a model which is built not using the jth data fold; n_j is the number of test points in the jth test set. It should be noted that, as it can be seen from Eq.(10), the CVErr is calculated using strictly only the test data.

The obtained approximation results are summarized in Table 2 and Table 3. Table 2 shows CVErr values averaged over all the eight responses for full-scale fuselage Design 1 and Design 2 as well as corresponding five and four stiffener panels. It is observed that in this study for all the responses the overall best approximation results were obtained using the ABFC technique. Table 3 shows averaged CVErr values of the ABFC for the individual responses. For the variable k_1 an error of about 1% is obtained however for all the other variables the error is around 9%. The elaborated ABFC models are used in the further studies described in Section 6.

Table 2: CVErr results averaged over all eight responses for full scale fuselages and corresponding panels

	FP	LWP	RBF	Kriging	MARS	SVR	ABFC
Design 1 full-scale structure	7.64	7.53	8.09	8.68	8.51	8.09	6.75
Design 1 four-stiffener panel structure	8.04	8.72	9.94	8.63	8.94	8.38	7.75
Design 2 full-scale structure	11.03	12.46	11.32	10.39	12.24	10.93	9.95
Design 2 five-stiffener panel structure	9.93	10.27	13.00	10.03	9.76	9.57	8.71

Table 3: CVErr results for the individual variables approximated by ABFC (averaged over all four designs)

	k_1	k_2	P_1	P_2	P_3	u_1	u_2	u_3
CVErr	0.91	8.74	9.02	9.41	7.88	9.57	10.90	9.89

An example of a comparison of load-shortening curves of metamodels developed using the ABFC versus curves of numerical simulations with FEM analysis and linear piece-wise simplification is shown in Figure 5. By comparing these curves, it is noticed that metamodels usually are more conservative than actual FEM analyses. This can be outlined as advantage if the metamodels are used for preliminary design of stiffened structures.

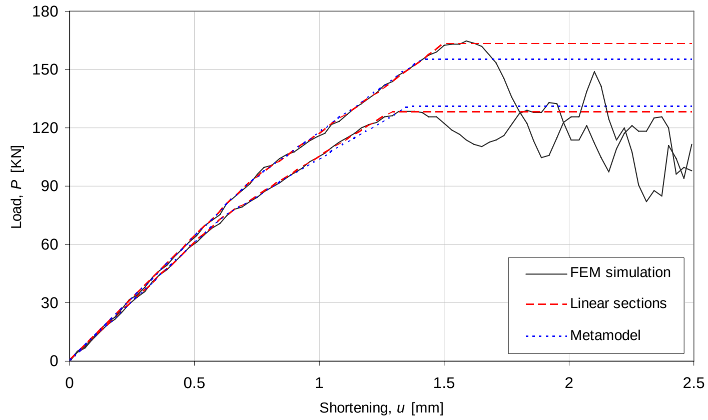

Figure 5: Material softening in three-stiffener design influence over numerical load-shortening curves

6. Results

The metamodels are incorporated into the optimisation procedure with dual aim. First aim was to estimate the scaling factor between the panel design and the full-scale structure and the second aim was to derive Pareto frontiers and optimum solutions which could be used in elaboration of the optimum design guidelines.

6.1 Estimation of the Scaling Factor between the Full-scale and the Panel Structure

One of the principal research aims was to estimate the scaling factor *C* between the full-scale structures and the panel designs. It should be noted that the two designs considered had even number (Design 1) and odd number (Design 2) of stiffeners. Nevertheless, by averaging the domain of interest for both designs the estimated scaling factors were relatively similar (Table 4). However, Design 1 had lower standard deviation thus it represents a lower parametrical sensitivity. Also it should be noted that the scaling factors may also be estimated using approximation models depending on the four design variables and using the scaling factors as responses. Such procedure would provide a much higher accuracy than the simple average value used here – in Figure 6 and 7 the changes in the scaling factor depending on the design variables are clearly pronounced.

Table 4: Average scaling factors and their confidence intervals

	k_1	k_2	P_1	P_2	P_3	u_1	u_2	u_3
Design 1	0.70 ±0.03	0.69 ±0.11	1.41 ±0.19	1.39 ±0.24	1.41 ±0.24	0.16 ±0.08	0.16 ±0.08	0.16 ±0.10
Design 2	0.62 ±0.04	0.77 ±0.11	1.49 ±0.41	1.45 ±0.37	1.35 ±0.35	0.22 ±0.09	0.19 ±0.09	0.18 ±0.09

By graphical validation in Figure 6 and Figure 7 one could notice that both designs tend to have similar scaling factor dependencies. Thus also the scaling factor transition between the number of stiffeners in the structure and load carrying capacity can be extracted. By comparing the dependency from number of stiffeners *N* versus the panel length *L* and the height of the stiffeners *h* it is obvious that Design 2 with the relatively narrower flange step is more sensitive to the stiffener total height. Furthermore, in the case of low number of stiffeners the scaling factor tends to diverge, this could be explained by possible evolvement of a different post-buckling mode shape pattern.

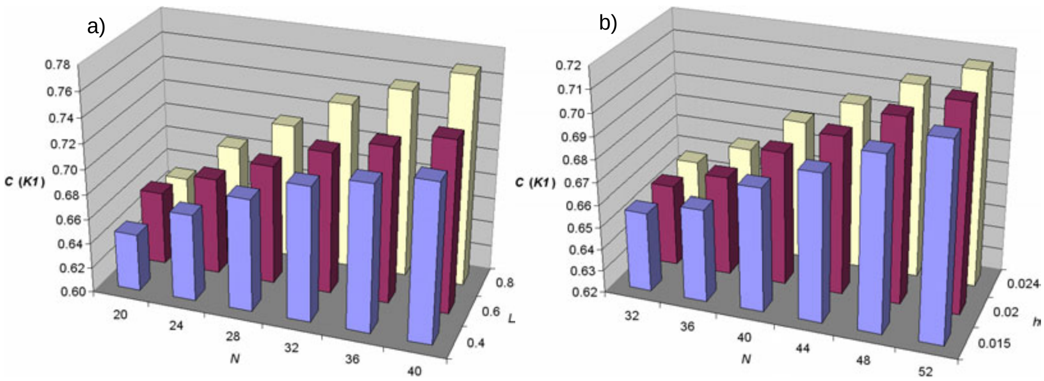

Figure 6: Scaling factor (pre-buckling stiffness k_1) bar charts for Design 1
a) R = 0.6 and h = 0.02; b) R = 1.0 and L = 0.6

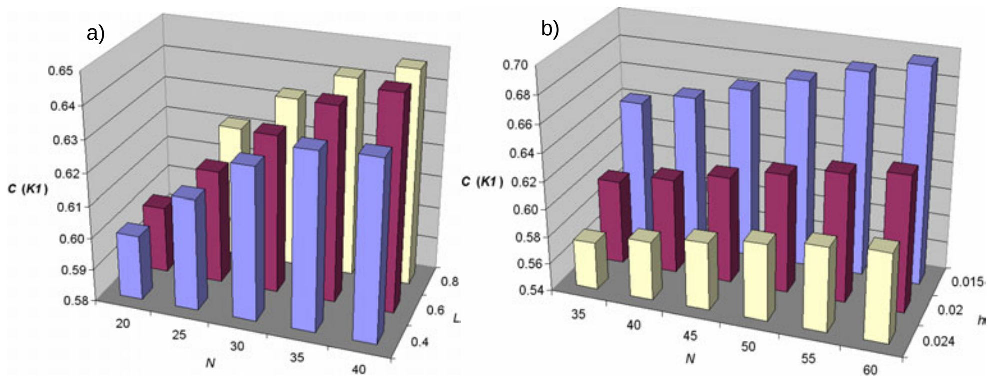

Figure 7: Scaling factor (pre-buckling stiffness k_1) bar charts for Design 2
a) R = 0.6 and h = 0.02; b) R = 1.0 and L = 0.6

6.2. Optimisation

The full domain of possible response characteristics for both designs of the full-scale structures have been evaluated by forming the cloud-type representations for combinations of the possible response values as show in Figure 8. The skin buckling load P_1 and the post-buckling reserve ratio P_2/P_1 have been elaborated versus the total volume V_{tot} of the full-scale structure or the pre-buckling stiffness k_1.

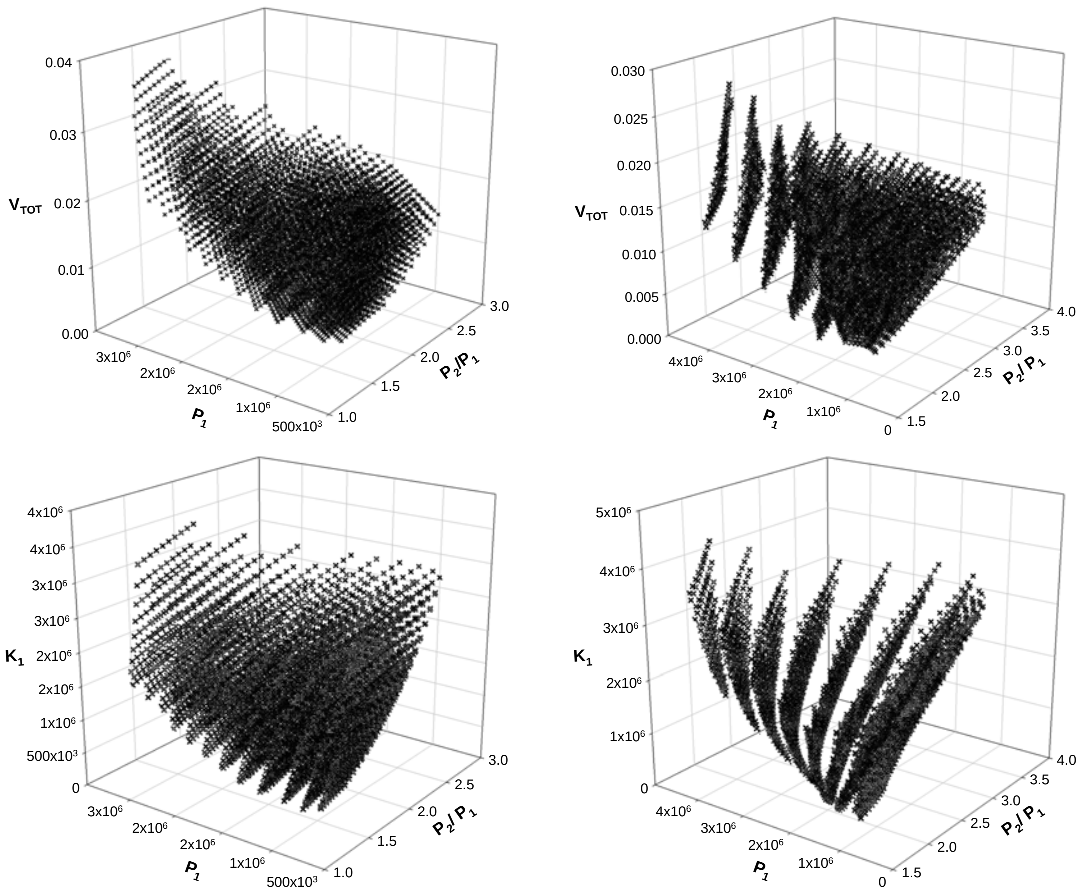

Figure 8: Resulting clouds of combinations of response values for Design 1 (upper two) and Design 2 (lower two)

Here the combinations of the minimum values per each dimension represent Pareto-optimal solutions. They have been further elaborated to create Pareto-optimal fronts (Figure 9) for the optimum design guidelines. Furthermore, it may be stated that the results form dense clouds of results so that there are wide design variety to achieve alternative structural qualities without almost any weight or performance penalty.

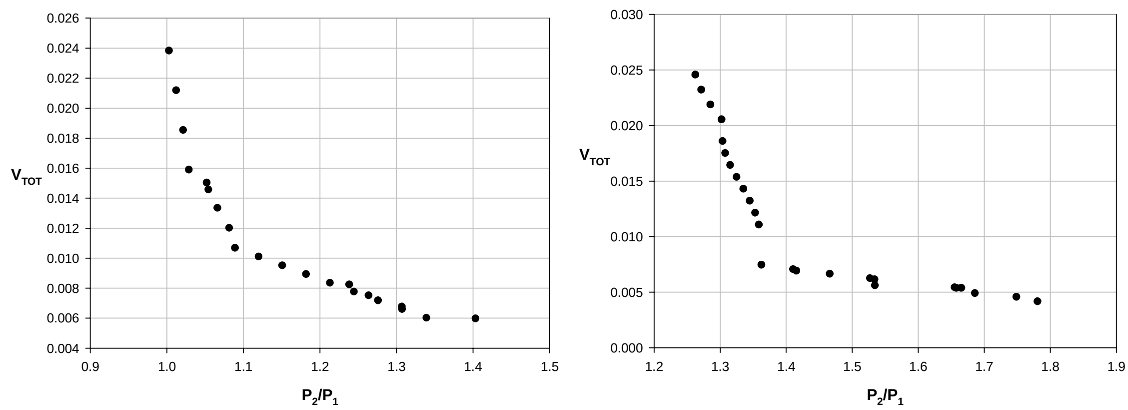

Figure 9: Pareto-optimal fronts for total volume V_{tot} versus post-buckling reserve ratio P_2/P_1 for Design 1 (on the left) and Design 2 (on the right)

Moreover, it may be generalised that most variety of design choices are supported mainly for the relatively low load carrying capacity, thus design optimisation including the post-buckling reserve ratio would be a reasonable way for further decrease of the structural weight in composite stiffened structures. It also may be stated that designs with high number of stiffeners – having additional volume penalty, tend to have high pre-buckling stiffness and strength meanwhile narrowing the post-buckling reserve ratio.

7. Conclusions

It was shown that the methodology based on the metamodeling of the load-shortening response dividing it into three piece-wise linear sections can be elaborated for the fast simulation practice for preliminary design of curved stiffened panels. It is concluded that the elaborated metamodels are efficient in surrogating the FE analysis of the different considered stiffened structures. For the particular metamodeling tasks the Adaptive Basis Function Construction approach of sparse polynomial construction gave the most accurate metamodels – for all the variables except the pre-buckling stiffness variable k_1 a cross-validation relative error of about 9% is obtained while for the k_1 the error is about 1%.

Moreover it was demonstrated that the acquired metamodels can be utilized for extracting of the transition scaling factor between the full-scale structures and the stiffened curved panel designs without the compromising the preliminary design reliability.

Also the full domain of possible response characteristics from the full-scale stiffened structure Design 1 and Design 2 have been elaborated in order to estimate the volume or structural stiffness dependencies versus skin buckling load and post-buckling reserve ratio. Such a procedure allows the designer to identify the parametrical sensitivities and guides for the structural weight savings. Additionally also Pareto-optimal fronts have been elaborated for the full-scale composite structures estimating the design configurations applicable for elaboration of the optimum design guidelines.

It should be noted that the resulting design procedure is more than 1000 times faster than FE design and provides an effective optimisation tool for the preliminary study of composite stiffened shells in addition to optimum weight design.

8. Acknowledgements

This work was partly supported by Latvian Council of Science grant 09.1262 and Riga TU grant ZP-2008/3.

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