Shell Structures: Theory and Applications, Vol. 2 – Pietraszkiewicz & Kreja (eds)

Metamodels in optimisation of plywood sandwich panels

K. Kalnins
Riga Technical University, Institute of Materials and Structures, Riga, Latvia

G. Jekabsons
Riga Technical University, Institute of Applied Computer Systems, Riga, Latvia

K. Zudrags
Latvian University of Agriculture, Department of Wood Processing, Jelgava, Latvia

R. Beitlers
Riga Technical University, Institute of Materials and Structures, Riga, Latvia

ABSTRACT: In this paper, the strength and the weight effectiveness for the plywood I-core sandwich structure is elaborated. For the plywood sandwich concept the plywood sheets are used at the outer surfaces to maximize rigidity while introducing in between adhesively bonded plywood stiffeners to keep the whole sandwich structure together. The structural stiffness capacity and weight efficiency are elaborated for the pure plywood structure and the I-core sandwich panel with the corresponding height and width dimensions. A metamodelling procedure is applied by approximating the finite element stiffness response values with parametrical functions employing the Adaptive Basis Function Construction approach. The resulting design procedure provides an effective optimal design tool that enables to estimate the stiffness and the weight optimum solutions.

1 INTRODUCTION

Typical sandwich panels, which are made of lightweight core and two face sheets are increasingly substituting commonly used plate structures in the aerospace, naval, transportation, and building industry. Dramatic advances in sandwich composite technology is a result of their superior qualities in terms of weight-to-strength ratio, high stiffness, ease of manufacturing, acoustic and thermal insulation, repair capability, and flexibility in design applications.

It has been generalised that enlargement in the core height increases the stiffness of the sandwich without substantially increasing weight or cost (Zenkert 1997). It is assumed that plywood industry could profit by replacing thick-wall panels with the thin-wall sandwich structures. One of the strategies would be to build the plywood sandwich structure by utilising the I-core type stiffeners made from plywood strips. By utilising the most common plywood thicknesses the complexity and the manufacturing costs could be reduced to meet the requirements for the mass production.

In (Kalnins et al. 2006) an optimisation procedure involving metamodelling is applied for laser welded I-core and V-core sandwich structures for naval applications providing the cost/weight effective design solutions. More recent research (Kalnins et al. 2008a) involving Os, C, Z and Oc – type cores have confirmed the effectiveness of metamodelling for the sandwich panel design procedure.

In this paper an optimisation procedure is proposed for plywood I-core sandwich panels. The objective was to establish valid approximations in design optimisation of sandwich shell structures.

2 METAMODELLING PROCEDURE

In many different industrial applications, to cut down the computational cost of complex, high fidelity scientific and engineering simulations, metamodels, also referred to as surrogate models, are constructed that mimic the behaviour of the simulation models as closely as possible while being computationally cheap(-er) to evaluate (Chen et al. 2006, Kalnins et al. 2006, Kalnins et al. 2008b). The process of design optimisation involving metamodelling usually comprises three major steps which may be interleaved iteratively: 1) sample selection (known as design of experiments); 2) construction of the metamodel that best describes the behaviour of the problem and estimation of its predictive performance; 3) employment of the metamodel in the optimisation task, i.e., finding the best values for input variables with which the system achieves the optimum response.

Originally metamodelling was associated with low-degree polynomial regression models which have global nature in describing numerical responses. They have been well accepted in engineering practice, as they require low number of sample points and are computationally very efficient. On other hand they are loosing efficiency when highly nonlinear behaviour should be approximated. Instead, higher-degree polynomials can be employed. However, if no special care is taken, they tend to overfit the data and produce high errors especially in regions where the sample points are relatively sparse.

One possible remedy for the overfitting problem is employment of the subset selection techniques. These are aimed to identify the best (or near best) subset of individual polynomial terms (basis functions) to include in the model while discarding the unnecessary ones, in this manner creating a sparse polynomial model of increased predictive performance.

However the approach of subset selection assumes that the chosen fixed full set of user-predefined basis functions (usually predefined just by fixing the maximal degree of a polynomial) contains a subset that is sufficient to describe the target relation sufficiently well. Hence the effectiveness of subset selection largely depends on whether or not the predefined set of basis functions contains such a subset.

In this study a different sparse polynomial model building approach is used – Adaptive Basis Function Construction, ABFC (Jekabsons 2008, Jekabsons & Lavendels 2008). The approach enables generating sparse polynomials of arbitrary complexity and degree without the requirement to predefine any basis functions or to set the degree – all the required basis functions are constructed adaptively specifically for the data at hand. Additionally, in contrast to a number of other state-of-the-art metamodelling techniques the models built by the ABFC can be expressed as explicit and simple-to-use regression equations.

Assuming that x is an input to the actual computer analysis or natural test, generally a polynomial regression model can be defined as a basis function expansion:

$$F(x) = \sum_{i=1}^{k} \beta_i f_i(x) \tag{1}$$

where β = coefficients of the model; k = the number of the basis functions included in the model; and $f_i(x)$ = a basis function generally defined as a product of original input variables each with an individual exponent:

$$f_i(x) = \prod_{j=1}^{d} x_j^{r_{ij}} \tag{2}$$

where d = the number of input variables; and $\mathbf{r}$ = a $k \times d$ matrix of non-negative integer exponents such that r_{ij} is the exponent of the j-th variable in the i-th basis function. Note that when for a particular basis function all the exponents are equal to zero, the basis function is the intercept term. The coefficients β are determined by minimizing least squares:

$$\beta = \arg\min_{\beta} \sum_{i=1}^{n} \left(F(x_{(i)}) - y_{(i)} \right)^2 \tag{3}$$

where n = the number of available sample points; $x_{(i)}$ = the input value of the i-th sample point; and $y_{(i)}$ = the actual response value of the i-th sample point.

The matrix $\mathbf{r}$ completely defines all the basis functions in the model – each row corresponds to one basis function with all of its exponents. Construction of the model is carried out in an iterative manner directly with $\mathbf{r}$ using a set of simple so-called model refinement operators enabling adding, copying, modifying, and deleting the rows of $\mathbf{r}$, i.e. adding, copying, modifying, and deleting the basis functions of the model. A more complete discussion on the ABFC is given in (Jekabsons 2008). The ABFC approach, together with a number of other metamodelling techniques, is implemented in the VariReg software tool (Jekabsons 2009).

To evaluate predictive performance of the metamodels, in this study a 10-fold Cross-Validation (CV) method was used in which the full data set is divided in 10 equally-sized subsets. In each of the 10 CV iterations nine of the subsets are used for model building and one left subset is used as an independent test data set for evaluation of the built model. As a model accuracy measure the Relative Root Mean Square Error (RRMSE) was used:

$$RRMSE = 100\% \frac{1}{10} \sum_{j=1}^{10} \left(\frac{1}{SD_j} \sqrt{\frac{1}{|t_j|} \sum_{i \in t_j} \left(F_j(x_{(i)}) - y_{(i)} \right)^2} \right) \tag{4}$$

where t_j contains the indexes of the test samples in the test set of the j-th CV iteration and $|t_j|$ denotes the number of test samples in this data set; $F_j(.)$ denotes the model built without using the t_j samples; and SD_j = the standard deviation in j-th test set:

$$SD_j = \sqrt{\frac{1}{|t_j|} \sum_{i \in t_j} \left(y_{(i)} - \bar{y}_j \right)^2} \tag{5}$$

where $\bar{y}_j$ = the mean value of response in the j-th test set. The RRMSE measure shows how good the predictive performance of the built model is in comparison with the performance of a constant value.

3 CASE STUDY

3.1 *Design considerations*

Pure plywood panel versus I-core plywood sandwich have been modelled (Fig. 1) and analysed using ANSYS 4-node shell element SHELL 181. The finite element mesh size – 10 mm is taken constant for both design cases. Only one sandwich section with symmetrical transverse boundary conditions with

Shell Structures: Theory and Applications, Vol. 2 – Pietraszkiewicz & Kreja (eds)

corresponding panel section is modelled, thus design is not restricted to any particular plywood manufactured plate width.

The length of the panel $L = 1100$ mm and the distance between the load units $L_1 = 300$ mm are kept constant. The total height H and the width B are interconnected between the plywood panel and the sandwich structure. It was assumed that each ply has thickness of 1.4 mm both for panel and sandwich design. Moreover stacking sequence has been modelled assuming that each layer is perpendicular to the upper and lower one, thus the plywood always consists of an odd number of plies.

The stiffness responses from the four point bending test under the constant load $P = 1000$ N have been elaborated by extracting the global deflection value. The stiffness ratio ΔK is calculated as division of pure plate stiffness value Kp by corresponding sandwich plate stiffness Ks value extracted from the numerical analyses. The ΔK value indicates the stiffness increase or decrease of the sandwich concept versus pure plywood plate design. Another measurement extracted is weight efficiency ratio ΔW, which indicate the weight savings from the sandwich design concept.

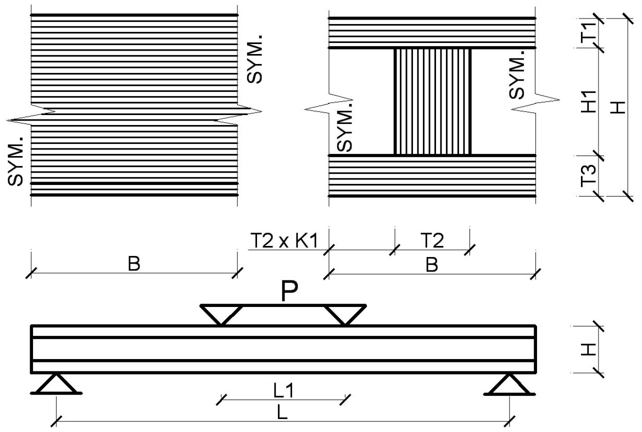

Figure 1. Design considerations sandwich panels.

3.2 *Design variables and constraints*

Five design variables (Fig. 1) are chosen for the sandwich panel design: the panel height – *H,* the number of plies in the upper sandwich plate – *T1*, the number of plies in the lower sandwich plate – *T3*, the number of plies for the I-core stiffener plate – *T2*. The stiffener spacing ratio is independent variable – *K1*, which directly influences the simulation section width – *B* for both panel and sandwich designs. The numerical values of the design spaces are outlined in Table 1.

Table 1. Design space

Notation	Lower bound	Upper bound	Units
T1	3	7	
T2	7	17	
T3	3	7	
K1	0.75	3	
H	27	40	mm

As the number of plies is an integer variable the design space is composed for each discrete level of four design parameters (*H*, *T1*, *T2*, *T3*), thus 324 sample points cover all combinations of the design space. Additionally eight levels of the stiffener spacing ratio (*K1*) variable are added for the fifth dimension.

Employing the ABFC metamodel construction technique twelve metamodels were constructed, so that for each of the six considered panel height H values there are two metamodels – one for the weight efficiency ratio ΔW and one for the stiffness ratio ΔK. The average RRMSE error for ΔW was 1.1% while for the ΔK the constructed models were always a perfect fit. Additional six metamodels were constructed for three level panel designs where the panel total width *B'* are considered constant (1250 and 2150). Here the average RRMSE error was 7.2%. Thus elaborated metamodels by they reliability have the capability to be used in further optimisation studies.

4 OPTIMISATION RESULTS

A Pareto optimisation problem is formulated where maximisation of the relative stiffness ratio ΔK is coupled with minimisation of the weight efficiency ratio ΔW. It could be assumed that the best performance could be reached when relative stiffness ratio tends towards the value of 1. Nevertheless the cost efficiency is directly linked with weight reduction. Thus dual strategies may exist for optimisation of sandwich performance: first to have sandwich panel stiffness close to the pure panel stiffness, which practically is weight inefficient, or second to have the sandwich height mach the stiffness while drastically reducing the total volume of the plywood.

A Pareto optimal front has been elaborated to evaluate the stiffness and weight effectiveness ratios for each panel height level. Comparison of the Pareto optimal fronts (Fig. 2) for different height plywood sandwich designs indicates the overall tendency that the highest stiffness ratio ΔK and the lowest weight efficiency ratio ΔW are reached for designs with the lowest heights. This confirms that major weight efficiency can be reached only for designs where the height of the panel is significant compared to the outer sandwich skins. Furthermore assessing the I-core stiffener width dependency from the number of plies – *T2* versus stiffness/weight efficiency ratio (Fig. 3) a general trend can be observed that increase in stiffener width compromises the stiffness ratio ΔK. It should be noted that increase in core stiffener width actually increases the panel width as these variables are interconnected.

The constant width *B'* Pareto optimal fronts have been elaborated (Fig. 4) constraining the designs so that only discrete number of stiffeners should be used. It can be observed that the Pareto-optimal front gradually falls back for the relative stiffness measure with each increase of the panel height *H* while preserving the trend meaning that for a constant weight efficiency ratio the load carrying capacity ratio can be tailored.

The elaborated metamodels applied in the optimum design guidelines provide efficient tool for tailored plywood sandwich panel design procedure.

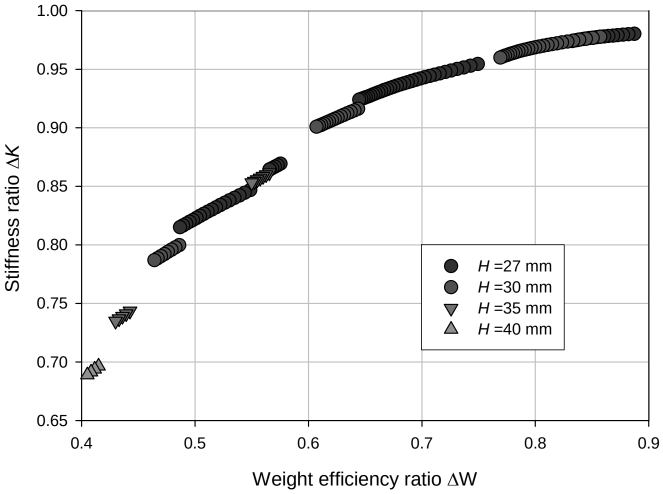

Figure 2. Pareto optimal front for different height plywood sandwich panel designs.

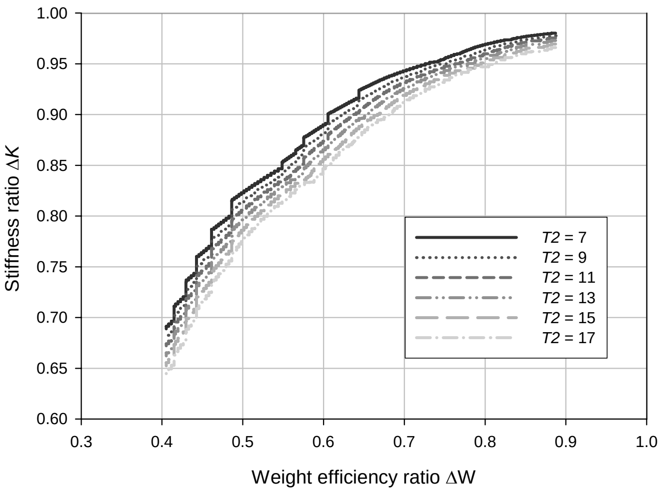

Figure 3. Pareto optimal fronts for different sandwich core thicknesses

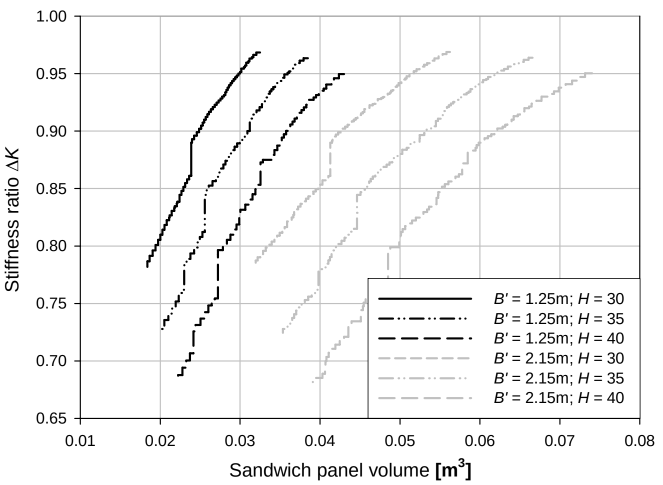

Figure 4. Pareto optimal front for specific panel width designs

5 CONCLUSIONS

The Pareto optimisation problem has been formulated and methodology based on metamodelling has been developed for the plywood sandwich panel stiffness and the weight efficient designs. Five design variables were considered and elaborated in numerical sampling strength analysis procedure by finite element code ANSYS. The optimisation results demonstrate the overall tendency that plywood sandwich structure versus pure plywood panel design could be nearly 60% weight efficient maintaining 70% of the load carrying capacity. Meanwhile maintaining the same design height the sandwich structure could reach nearly 97% of the pure panel stiffness with 10% weight savings. Furthermore it was shown that a great potential exists for plywood sandwich designs when the panel height is not a design constraint. Therefore I-core sandwich panels can be tailored to meet the stiffness requirements with significant decrease in material volume of the plywood used in manufacturing of the panel structures.

6 ACKNOWLEDGEMENTS

This work was partly supported by the Latvian Council of Science grant nr. 09.1262.

7 REFERENCES

Chen, V.C.P., Tsui, K-L., Barton, R.R. & Meckesheimer M. 2006. A review on design, modeling and applications of computer experiments. *IIE Transactions*, 38(4): 273-291.

Jekabsons, G. 2008. Ensembling adaptively constructed polynomial regression models. *Intelligent Systems and Technologies*, 3(2): 56-61.

Jekabsons, G. & Lavendels, J. 2008. Polynomial regression modelling using adaptive construction of basis functions. *Proc. intern. conf. IADIS applied computing, Algarve, Portugal*.

Jekabsons, G. 2009. *VariReg software tool, version 0.9.18*, (http://www.cs.rtu.lv/jekabsons/).

Kalnins, K., Eglitis, E., Jekabsons, G. & Rikards R. 2008a. Metamodels for optimum design of laser welded sandwich structures. In K.Jaramai and J.Farkas (eds), Design, fabrication and Economy of Welded Structures *Proc. intern. Conf DFE2008., Miskolc, Hungary, 24-26 April 2008*, Harwood Publishing: Chichester: 119-126.

Kalnins, K., Ozolins, O. & Jekabsons, G. 2008b. Metamodels in design of GFRP composite stiffened deck structure. *Proc. of 7th ASMO-UK/ISSMO intern. conf. on engineering design optimization, Association for Structural and Multidisciplinary Optimization in the UK, London, UK.*

Kalnins, K., Skukis, E., & Auzins, J. 2006. Metamodels for I-core and V-core sandwich panel optimisation. In W. Pietraszkiewicz & C. Szymczak (eds), *Shell Structures: Theory and Applications; Proc. 8th intern. conf., Jurata, Poland, 12-14 October 2005*, Taylor & Francis: London: 569-572.

Zenkert, D. 1997. *The Handbook of Sandwich Construction*. London: EMAS.