Metamodels for optimum design of laser welded sandwich structures

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Abstract
All-metal sandwich panels, made by a process of laser welding faceplates to core-stiffeners, show advanced cost/weight properties compared with the conventional structural applications of stiffened plates. However optimal design of these advanced structures requires a fast simulation procedure that should have the same level of reliability compared to finite element calculations and natural tests, while being more time effective and less complex. It was shown that different polynomial functions together with design of computer experiments can contribute to such an aim by providing simple however reliable metamodels. The validation procedure indicated an average of 10% relative root mean square error prediction accuracy, and due to this precision the procedure is capable to be used for further (cost/weight) design optimisation together with structural sizing studies and parametric sensitivity analysis.
Keywords: metamodelling, different core type sandwich panels.

1. Introduction

The development of new materials and new manufacturing techniques has accelerated during the last several years, and this has made an impact on innovative structural solutions introduced in industrial production. One of these new ideas is the laser welding technique, which has started to find increasing application among different methods of joining components of ship structures Roland (2006). Laser welding is one of the newest welding techniques, and has been available since the 60's. The main advantages of laser welding are low welding distortions, high productivity and easy automation, and these have opened new opportunities in the design of steel structures. The latest advances in sandwich structures compiled by researchers from the aerospace, wind turbine, marine, rail/road transport industries have been summarised recently by Shenoi et al. (2005).

All-metal sandwich panels, made by a process of laser welding of faceplates to core-stiffeners, show advanced cost/weight properties compared with conventional structural applications of stiffened plates. The main benefits of a sandwich structure are caused by the high stiffness and bending strength properties due to the location of the material as far as possible from the neutral axis of the panels Zenkert (1997).

Progress in sandwich structures has been enabled by the development of a straightforward and inexpensive manufacturing technology for different core types Wadley et al. (2003). Cores of interest include honeycombs Cote et al. (2004), pyramidal and tetrahedral trusses Chiras et al. (2002), as well as diamond ducts and corrugated prismatic cores Pokharel et al. (2005). Full-scale application requires that the structural performance be characterised using a combination of analytical and numerical results, validated by experiments. The structural analysis of sandwich panels with thin flat faces was undertaken as early as the 1940's, particularly for aeronautical applications. The theoretical foundation and governing differential equations for the analysis of sandwich panels were presented in detail by Allen (1969) and Plantema (1966). Design formulations for different core type all-metal sandwich panels filled with core material or empty, and with symmetric or asymmetric faceplates were recently summarised Romanoff & Vasta (2006) where relations necessary to calculate the stiffness and stress of sandwich panels were presented for application in an equivalent 2D for full 3D finite element (FE) analysis. This procedure of different core type sandwich design was implemented into the commercially available software code ESAComp. However a significant disadvantage compared to a full 3D FE analysis is the estimation accuracy of the total stresses. In real structures the total stress and strain would be the sum of local and global stresses, so neglecting these local stresses leads to underestimation in the presented analysis procedure with respect to the real structure under experimental testing.

Currently sandwich panels composed of I-core and V-core stiffeners are among the most extensively used in manufacturing, however other core-stiffener of Z-core, C-core, Osquare-core, Ocircle-core as seen in Figure 1, continue to retain interest for further investigation. The different core type panels represent different manufacturing and material supply strategies, which have a number of benefits including added value from innovative manufacturing or seamless welding joints if the stiffeners are joined through the core structure to the top plate.

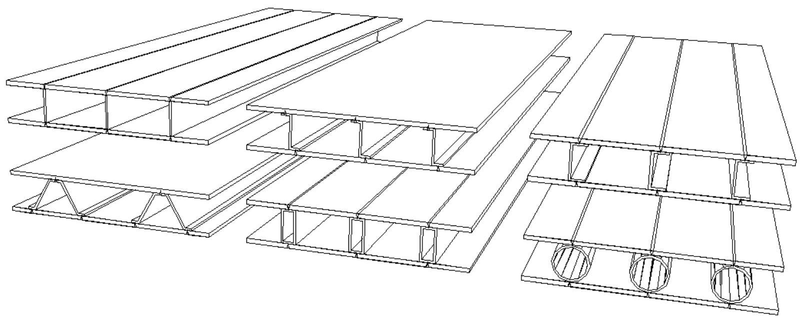

Figure 1. I, Z, C, V, Osquare and Ocircle core type sandwich panels.

2. Metamodelling

2.1 Design of computer experiments

The main issue related to metamodelling of structural responses is how to achieve good accuracy of approximated models with a reasonable number of sample experiments. When FE analyses are used to determine stress/strain responses the use of classical design of experiments (DOE), which needs repeated runs, is not effective. Instead deterministic computer experiments sampled according to the space-filling criteria, for example the Latin Hypercube (LH) design McKay et al. (1979), should be used as a basis for evaluation of parametric/non-parametric approximation functions. Typically LH design sample points tend to spread out to the corners of the unit cube, which can be avoided by introducing optimality criteria such as Audze & Eglajs (1977), Minimax and Maximin designs Johnson et al.(1990), Mean Square Error (MSE) and uniform designs Fang & Wang, (1994), Morris & Mitchell (1995), which is a generalisation of Eglajs' criterion. All these designs require pre-knowledge regarding the actual amount of experiments needed for fixed-size design, thus the sampled design space cannot be extended or narrowed without affecting the optimality criteria. Considering this, a more efficient strategy is proposed Auzins (2004), by arranging and adding new experimental points to an already existing design of experiments according to a space-filling criterion, thus achieving a good balance between the space filling quality in the whole design space and quantitative improvement by adding sample points. Moreover sequential designs can be obtained by adding new points to the already existing design space or by arranging the points in optimised large sample quantity design spaces. An advantage of the proposed approach is the fine sampling quality even before all experiment runs are performed, which once elaborated could be made publicly available (www.rtu.lv/mmd/).

2.2 Polynomials as approximation functions

Originally metamodelling was associated with low-order polynomial regression models which have global nature in describing numerical responses. They have been well accepted in engineering practice, as requiring low number of sample points, and are computationally very efficient. On other hand they are loosing efficiency when highly nonlinear behaviour should be approximated. Instead the higher-order polynomials can be employed however, if no special care is taken, they tend to overfit the data and produce high errors in regions where the sample points are relatively sparse. As a possible remedy for the overfitting problem, a subset selection (or model building) techniques (e.g., see Mayers & Montgomery (2002)) may be used. They are aimed to identify the best subset of polynomial terms (or basis functions) to include in the model and to remove the unnecessary ones, in this manner increasing model's predictive performance. However the approach of subset selection assumes that the chosen *fixed* full set of *predefined* (usually just by fixing the maximal order of the polynomials) basis functions contains a subset that is sufficient to describe the target relation sufficiently well. Hence the effectiveness of subset selection largely depends on whether or not the predefined set of basis functions contains such a subset. A short outline of another approach of adaptive construction of basis functions is described in the next subsection. It should be noted that the approach does not require the user to predefine a set of basis functions (or to set the maximal order of the polynomials) – instead the required basis functions are constructed automatically.

2.3 Adaptive basis function construction of polynomial metamodels

Generally a polynomial model can be defined by a linear summation of basis functions:

$$\hat{y} = \sum_{i=1}^{k} a_i f_i(x) \tag{1}$$

where k is the number of the basis functions included in the model (equal to the number of model's parameters); and $f(x)$ are the basis functions which generally may be defined as a product of the input variables each raised to some order:

$$f_i(x) = \prod_{j=1}^{d} x_j^{r_{ij}} \tag{2}$$

where r_{ij} is the order of the j-th variable in the i-th basis function (a non-negative integer). Note that when all r_j's of a basis function are equal to 0, we have the intercept term.

The Adaptive Basis Function Construction (ABFC) approach Jekabsons et al. (2007) allows generating polynomials of arbitrary complexity without the requirement to predefine any basis functions. In ABFC the standard model refinement operators of subset selection, namely addition and deletion of basis functions, are replaced with other operators, which not only allow adding or deleting basis functions but also allow changing the basis functions themselves (increasing and decreasing orders). Thus in ABFC the search operates directly with the matrix *r* in the Eq.2.
Still the refinement operators of ABFC allow using the same search algorithms as in subset selection – in Jekabsons et al. (2007) ABFC was used together with Sequential Floating Forward Selection proposed by Pudil et al. (1994). In order to achieve the trade-off between simplicity and predictive performance of models the Corrected Akaike's Information Criterion was used Hurvich & Tsai (1989).

2.4 Metamodel validation

Presented research focuses on validation of the selected approximating functions in metamodel building of sandwich structure stress/deformation responses. A total of five hundred sequential design sampling points has been elaborated for training the different core sandwich panel metamodels. The Cross-Validation (CV) technique has been used, where validation procedure has been applied with 400 training points and 100 validation points (5-fold CV). In order to assess the decrease in prediction performance a half of the sample points where selected for training and half for validation purpose (2-fold CV). The test sample accuracy measure used is the Relative Root Mean Square Error:

$$RMSE\% = 100\% \frac{\sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}}{STD} \qquad (3)$$

where y_i is *i*-th test point, $\hat{y}_i$ is predicted value of *i*-th test point, *n* is the number of test sample points, and STD is the standard deviation in test sample:

$$STD = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \bar{y})^2}{n}} \qquad (4)$$

It should be noted that RMSE% and STD are calculated using strictly only the test sample and averaged over the Cross-Validation runs.

3. Case Study

The present paper deals with derivation of metamodels for a fast simulation tool that should have the same level of reliability compared to FE calculations and natural tests, however required to be more time effective and less complex. Moreover the developed simulation procedure should be applicable for derivation of optimal design guidelines. A six different core type sandwich panels under bending loading were studied for application as deck panels in a modularised ship concept. Initial studies where metamodels for I-core and V-core type panels Kalnins et al. (2004) and Barkanov (2006) were used in design optimisation revealed explicit cost/weight efficiency for certain panel applications. The choice of design variables depended on the core type of all-metal sandwich panels and industrial demands. The geometrical design variables of all considered sandwich core types are shown in Figure 2. All core type stiffeners were similarly positioned in plates at a distance measured to the plate or core profile neutral vertical axis. Also, the V-core stiffener had a constant 60^0 opening angle, thus besides the spacing factor as used for the other core analysis a constant was added in order to avoid stiffener crossing.

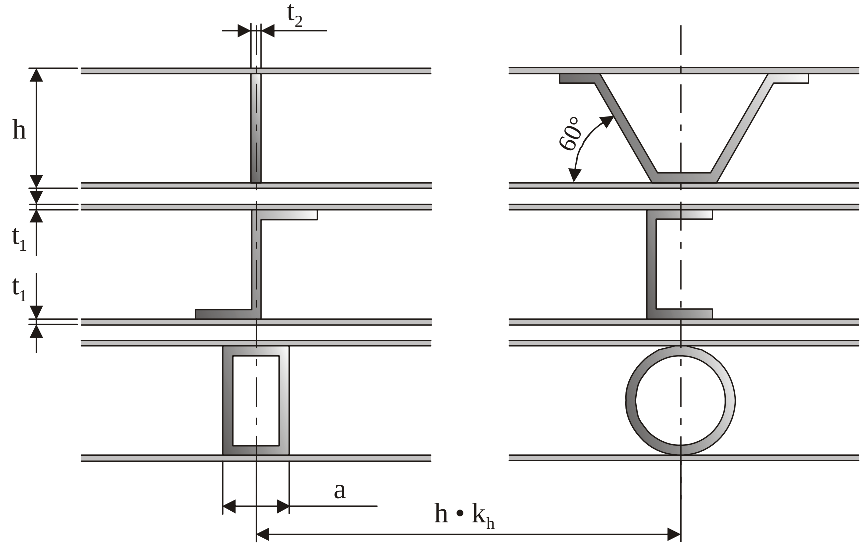

Figure 2. Geometrical parameters for different core type panels

A design process was conducted linking the width of the panel *B* with the symmetrical number of stiffeners *n* and the stiffener spacing parameter. Thus multiplying the panel height *h* and the core stiffener spacing factor k_h a stiffener spacing parameter can be established. Furthermore the panel length *L* parameter and two corresponding plate thicknesses are taken as design variables: t_1 – cover plate thickness and core stiffener thickness t_2. The full domain of interest representing lower and upper bounds of the design parameters is outlined in Table 1.

Table 1. Geometrical variables of different core type panels

Name	Notation	Design boundaries		Dimension
		Lower	Upper	
Panel length	L	3	7	m
Panel height	h	4	16	mm
Top and bottom plate thickness	t_1	2	4	mm
Core stiffener thickness	t_2	1.5	4	mm
Core stiffener spacing factor	k_h	1.5	4	
Symmetrical number of core stiffeners	n	2	6	

All-steel sandwich panel numerical experiments were conducted using FEM commercial software ANSYS employing SHELL 181 - 4-node shell element. Initial model verification was performed comparing deflection and stress results obtained in physical tests Kozak (2004). Simply supported boundary conditions were applied to the transverse edge bottom nodes corresponding to the boundaries conditions used in the testing rig. A combined loading has been applied in particular uniformly distributed pressure load of 3 kPa on the top plate and a concentrated load of 1 kN was applied in the centre of the sandwich panel. This corresponded to the load levels required for certification of deck designs corresponding to the DNV (2003) design guidelines.

4. Results

A cross-validation procedure has been carried out comparing different order of full polynomials and polynomials of adaptive basis functions. The prediction errors of the essential structural response metamodels have been compared. In particular the global deflection of the sandwich panel – *DEF_BOT*, the local deflection ratio between the upper and lower sandwich plates – *DEF_DIF*, the equivalent stresses at the upper cover plate – *EQV_TOP*, and the maximum shear stresses from the sandwich core stiffeners – *SHEAR*. Comparison of the prediction accuracy by six different core type panels is summarized in Tables 2 and 3, where 2nd, 3rd, and 4th order polynomials are compared with partial polynomials elaborated by means of ABFC approach. One can conclude that the partial polynomials can significantly improve the prediction accuracy compared to the conventional 2nd order polynomials, which are mostly associated with engineering problems of the response surface methodology. For example, the precision of the deflection responses could be improved by an order of magnitude compared to the 2nd order polynomials. In contrary improvement in the equivalent stresses and share stresses characteristics is less efficient. By analysing 5-fold and 2-fold CV results, it could be outlined that, by decreasing amount of the training points, the most decrease of the approximation performance is the property of 4th order full polynomials. In contrast the performance of lower order and partial polynomials reduced in average by only 1%.

Table 2. Metamodel validation accuracy with 5-fold CV

Polynomial	2nd order	3rd order	4th order	Adpt.	Core – type design		2nd order	3rd order	4th order	Adpt.
Response	RMSE%						RMSE%			
DEF_BOT	35.51	20.51	13.27	1.73	I -core	C-core	33.94	18.25	11.04	1.45
DEF_DIF	29.75	13.06	5.73	1.21			31.07	14.22	5.94	1.56
EQV_TOP	17.43	9.57	9.99	7.54			18.11	10.63	13.08	8.71
SHEAR	12.53	8.10	10.52	6.69			11.96	6.60	7.94	5.21
DEF_BOT	33.15	17.22	9.50	2.72	Z-core	V-core	33.09	17.48	10.89	4.17
DEF_DIF	31.32	13.38	5.90	1.57			37.49	18.29	16.12	3.34
EQV_TOP	20.57	13.51	17.03	12.08			38.98	38.75	60.27	35.42
SHEAR	12.06	7.28	8.65	6.45			18.28	13.09	13.35	11.63
DEF_BOT	34.62	18.67	11.79	1.72	Os-core	Oc-core	37.36	21.79	16.18	3.41
DEF_DIF	34.02	15.18	7.10	1.62			30.87	14.34	6.45	1.40
EQV_TOP	18.08	10.29	12.40	8.82			20.04	11.40	11.22	7.83
SHEAR	15.31	9.24	11.40	8.06			22.14	15.77	22.31	15.79

Table 3. Metamodel validation accuracy with 2-fold CV

Polynomial	2^{nd} order	3^{rd} order	4^{th} order	Adpt.	Core – type design		2^{nd} order	3^{rd} order	4^{th} order	Adpt.
Response	RMSE%						RMSE%			
DEF_BOT	37.73	22.28	22.05	4.01	I -core	C-core	35.01	19.27	19.20	2.48
DEF_DIF	30.33	13.74	9.88	1.28			31.10	14.18	10.12	1.57
EQV_TOP	17.76	10.62	22.39	8.24			18.34	11.42	26.29	10.12
SHEAR	12.19	8.38	20.85	7.02			11.82	7.19	17.26	6.66
DEF_BOT	34.94	18.83	19.01	3.39	Z-core	V-core	34.09	17.98	21.03	7.25
DEF_DIF	31.19	14.26	10.56	1.80			38.01	18.58	13.21	3.92
EQV_TOP	20.26	13.88	30.42	11.81			44.97	49.12	91.77	46.29
SHEAR	12.15	7.35	18.69	6.62			19.13	15.39	25.88	13.47
DEF_BOT	35.05	19.13	18.93	2.47	Os-core	Oc-core	39.87	24.36	27.32	6.11
DEF_DIF	34.36	16.19	11.57	1.48			30.88	14.25	10.47	1.44
EQV_TOP	19.16	11.37	26.02	9.18			18.76	11.47	23.72	8.28
SHEAR	15.20	10.52	24.18	10.24			22.05	17.78	38.22	17.38

Conclusion

It was concluded that the elaborated metamodels of adaptive basis function construction as different parametrical polynomials are efficient in surrogating FE analysis of different core type sandwich structures. The approximations obtained, by their precision, are capable of serving in the development process for design guidelines of new sandwich or different composite structures. Moreover evaluated metamodels will be used for further (cost/weight) design optimisation together with structural sizing studies and parametric sensitivity analysis.

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