

# Metamodels in design of GFRP composite stiffened deck structure

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**This paper deals with the comparison of parametric and non-parametric metamodels used in design procedure of pultruded glass fibre reinforced plastic (GFRP) deck panels. Stiffened deck structure loaded under the three-point bending and uniform distributed load has been examined. A set of five hundred sample points has been evaluated by means of finite element analysis and stiffness and strength responses extracted for building of metamodels. Moreover, set of complimentary physical experiments has been carried out in order to verify the numerical model. Both advantages and disadvantages of parametric and non-parametric metamodels have been drawn for a specific engineering application.**

## I. Introduction

COMMON optimum design practice of stiffened structures under any loading combinations should involve extensive finite element analysis with complimentary experimental validation. However, such a procedure is only partly efficient as besides trade-off design the designer is seeking for alternatives in the overall perspective. Despite the advances in rapidly growing computational capacity, as in High Performing Computing or in GRID technologies, the enormous computational cost of complex engineering simulations makes it impractical to rely exclusively on simulation for the purpose of design optimization<sup>1</sup>. As a good practice, one could use mathematical approximations instead of full scale analyses, thus reducing the level of numerical optimization complexity. Metamodels, also called surrogate models, are constructed from response approximations extracted from actual simulation models. In particular, for determination of the most suitable metamodeling technique fitting to deck under the bending load design procedure a different parametric and non-parametric approximations have been compared – low order global polynomials, locally weighted polynomials, partial polynomials, and Multivariate Adaptive Regression Splines.

Important research issue associated with metamodeling is how to achieve good accuracy of a metamodel with reasonable number of sample points. The sampling techniques, often referred to as design of experiments, should be implemented to reduce the number of simulation runs without decreasing the accuracy of the metamodel. The differences between sampling strategies for physical experiments and for computer experiments should be noted. Whilst physical experiments have statistical experimental errors, numerical analyses are deterministic and results are obtained with 100% repetition and no statistical variance of model parameters<sup>2,3,4</sup>. Currently, there is a wide range of literature concerning different methods for DoCE<sup>5</sup>, which include many approaches for space-filling designs. It should be noted that the first space-filling design criterion<sup>2,3</sup> for numerical experiments was proposed at Riga Technical University by Audze and Eglajs. While the accuracy of a metamodel is directly related to the approximation technique used and to properties of the problem itself, the type of sampling approaches<sup>6</sup> have a direct influence on the approximation performance. It is generally accepted that space-filling designs, for example the Latin Hypercube design, are preferable for building of metamodels. In the current study, to reduce the number of computations, experimental design optimized according to Mean Square Error (MSE) criterion<sup>7,6</sup> has been selected.

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## II. Case study of pultruded GFRP deck structure

Among many other composite materials, the glass fibre reinforced polymer (GFRP) composites has been demonstrated to have great success in bridge engineering applications. It can be generalized that characteristics of a composite elements or structures can be tailored and designed to meet any desired specifications. The favorable characteristic of composite bridge decks are a high strength to weight ratio efficiency and relatively low material and manufacturing costs. There have been already developed and installed different deck systems with comparatively small spans up to 10 m<sup>8</sup>, however span lengths of GFRP deck applications are foreseen to expand over the next years. Most of the GFRP bridges constructed up to now<sup>9</sup> used multi-cellular pultruded deck systems. In principle, two construction forms are used: multi-cellular deck panels from adhesively bonded pultruded shapes and sandwich panels with different core structures<sup>10,11</sup>.

In present work, design of GFRP composite stiffened panel structures has been considered similar to ones currently manufactured by Rishon-Inter.Ltd (<http://www.rishon-inter.lv>). Eleven I-type stiffener deck design, as shown in Figure 1, has been elaborated in parallel with experimental tests performed at Riga Technical University, Institute of Materials and Structures using dedicated test equipment. Three-point bending test case has been considered for numerical analyses and physical tests along with uniformly distributed load test case that has been elaborated only numerically.

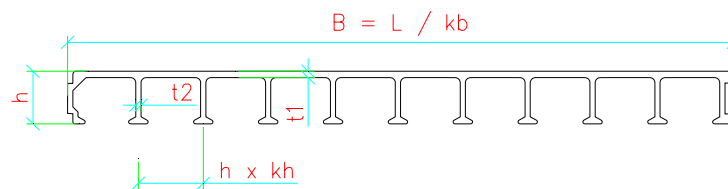


Figure 1. GFRP Deck geometry with parametrical variables

The flexural stiffness response of GFRP pultruded deck structure has been evaluated numerically by finite element method commercial software ANSYS<sup>12</sup> employing SHELL 181 4-node shell element. The mechanical properties for glass fibre composite used for full-scale analysis have been extracted by small coupon tests in tension and bending. Numerical values of load-deformation curve, stress, and strain distribution over the tested deck structure were extracted and incorporated in design of strain gauge locations for experimental validation. The numerical deflection and stress graphs of three-point bending test are shown in Figure 2. A comparative study between numerical and experimental results will be given in the chapter III.

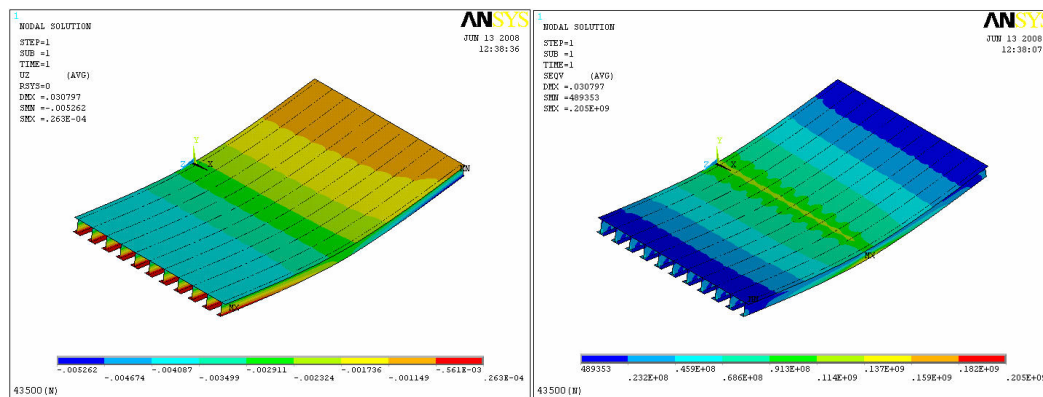


Figure 2. Three point bending numerical test – deflection (left) and stress-distribution (right) graphs

The choice of design variables should represent all geometrical parameters in optimum design procedure. However most of those variables are rationally interconnected. Exploration of non-rational variable combination mostly leads to relatively high approximation errors. In particular, when non-proportional ratios of span and height are used for bending problems this usually causes singularity in approximations. Therefore geometrical design variables with corresponding ratio variables have been proposed<sup>11</sup> for metamodeling procedure. As geometrical

variables the panel length parameter  $L$  and the panel height parameter  $h$  along with two plate thicknesses have been taken: the cover plate thickness  $t_1$  and the stiffener thickness  $t_2$ . Moreover, rational design variables as  $kb$  (ratio between the panel length and width) and  $kh$  (stiffener spacing parameter ratio between I-stiffener foot width and panel height) have been proposed. Such a procedure is required to restrain the combination where stiffener spacing is narrower than the deck height  $h$ . The numerical bounds of design variables are given in Table 1.

**Table 1. Design space for deck structure**

Name and notation		Lower bound	Upper bound	Units
Deck length	$L$	0.6	4	m
Deck stiffener height	$h$	0.03	0.10	m
Plate thickness	$t_1$	0.003	0.006	m
Stiffener thickness	$t_2$	0.003	0.006	m
Deck length to width ratio	$kb$	1.5	3	
Stiffener spacing ratio	$kh$	2	3	

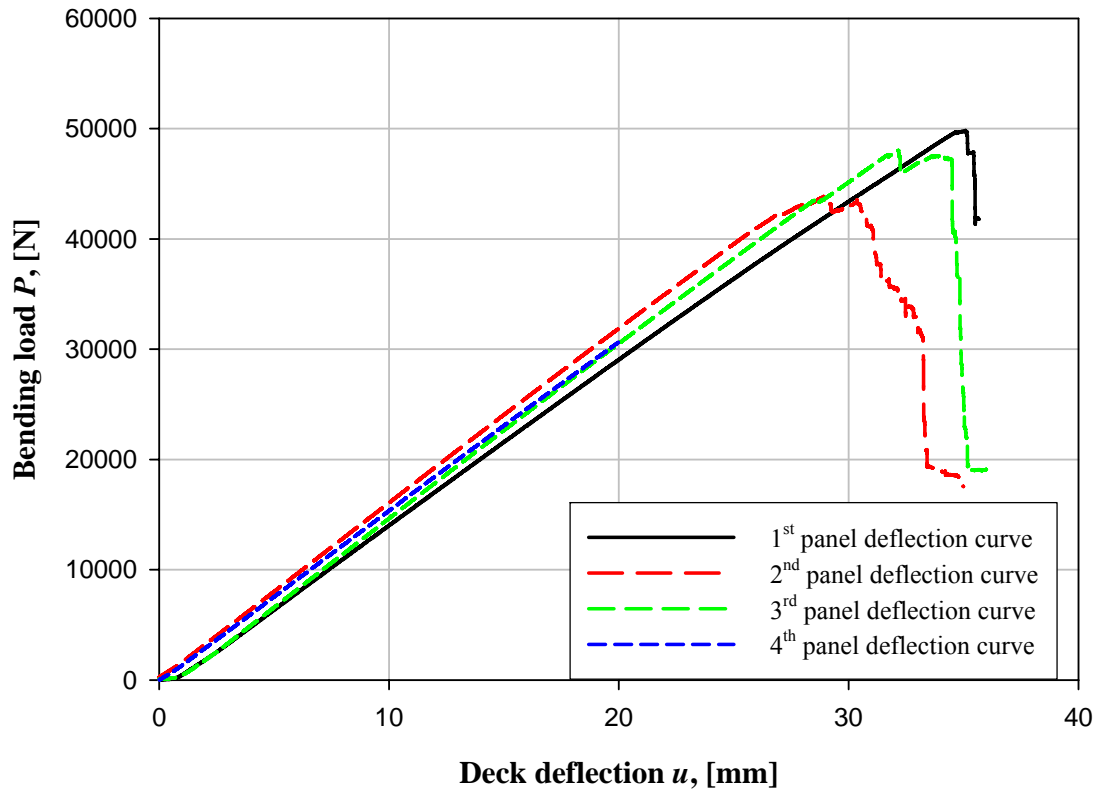
In order to achieve the best performance (minimal prediction error of metamodels), a space-filling design of five hundred sample points optimized according to the Mean Squared Error<sup>6,7</sup> uniformity criterion has been selected. A numerical sampling procedure involves a large amount of analyzed numerical data that are unacceptably time demanding. Therefore, all numerical data sets have been analyzed in parallel, exploring the LatvianGrid (<http://grid.lumii.lv>) computing capabilities, thus reducing the necessary time and outsourcing the computational capacities.

### III. Physical experiments

Three three-point bending tests until collapse of the structure for real deck panels have been performed. Adding the validation test that has been outfitted with strain-gauges and loaded until 60% of the total collapse load. The experimental setup of a one-meter span length and corresponding deflected collapse mode shape is shown in Figure 3. During the tests, the load-versus-deflection curves (outlined in Figure 4) and strains have been recorded by means of load cell and the strain gauge readings. Thus the data from physical experiments has been implemented in validation procedure of ANSYS<sup>12</sup> finite element model as summarized in Table 2.



**Figure 3. The experimental three-point bending test setup and collapse mode of the stiffened deck structure**



**Figure 4. Load-versus-deflection curves obtained experimentally**

The test results obtained experimentally for four tested deck structures have been compared with values obtained numerically by ANSYS<sup>12</sup>. One could observe from the Figure 4 and the Table 2 that all panels have practically the same loading stiffness. However, there is a certain divergence between obtained critical load levels. Nevertheless, in validation procedure the numerical deflection results have about 10% discrepancy with physical test results what should be considered as a good agreement between actual (manufactured) and numerical model. Moreover, numerical stress threshold value practically corresponds to the value obtained by small specimen tension tests. Validation procedure outlined that the load level corresponding to deflection limit legislated by building codes [1/250 to 1/150 of the deck span] are practically one fifth of the ultimate stress value.

**Table 2. Validation between experimental and numerical results**

Load(N)	1 <sup>st</sup> panel test, deflection (mm)	2 <sup>nd</sup> panel test, deflection (mm)	3 <sup>rd</sup> panel test, deflection (mm)	4 <sup>th</sup> panel test, deflection (mm)	ANSYS, deflection (mm)	ANSYS stresses (MPa)
5000	4.1	3.1	4.1	3.4	3.5	23.5
10000	7.4	6.3	7.1	6.6	7.1	47.1
15000	10.7	9.4	10.2	9.8	10.6	70.6
20000	14.0	12.5	13.4	13.0	14.2	94.1
25000	17.3	15.6	16.5	16.3	17.7	118
30000	20.6	18.8	19.7	19.6	21.2	141
35000	24.1	22.1	22.9	N.A.	24.8	165
40000	27.5	25.3	26.2	N.A.	28.3	188
43500	30.1	30.4	28.5	N.A.	30.1	205
48000	33.4	-	32.1	N.A.	34.0	226
49800	35.1	-	-	N.A.	35.3	234

#### IV. Employed metamodeling techniques

This section briefly overviews the employed metamodeling techniques: full global polynomials and locally weighted polynomials of 2<sup>nd</sup>, 3<sup>rd</sup>, and 4<sup>th</sup> order, Multivariate Adaptive Regression Splines (MARS), and partial polynomials constructed using Adaptive Basis Function Construction (ABFC) approach.

As described by Simpson et al.<sup>13</sup>, it is assumed that the inputs to the actual computer analysis are supplied in matrix  $\mathbf{x}$ , and the outputs (or responses) from the analysis – in vector  $\mathbf{y}$ . Then the true computer analysis code evaluates

$$y = g(x) \quad (1)$$

where  $g(x)$  is a complex engineering analysis function. The computationally efficient metamodel approximation is

$$\hat{y} = f(x) \quad (2)$$

such that

$$y = \hat{y} + \varepsilon \quad (3)$$

where  $\varepsilon$  includes both approximation and random errors.

##### A. Full global polynomials

Low-order polynomials are the most widely used metamodels<sup>1,13,14</sup>. For example, second-order polynomial can be defined as follows:

$$\hat{y} = \beta_0 + \sum_{i=1}^d \beta_i x_i + \sum_{i=1}^d \sum_{j=i}^d \beta_{ij} x_i x_j \quad (4)$$

where  $d$  is the number of input variables;  $\beta_0, \beta_i, \beta_{ij}$  are coefficients usually determined by the ordinary least squares method minimizing

$$\beta = \arg \min_{\beta} \sum_{i=1}^n (\hat{y}_{(i)} - y_{(i)})^2 \quad (5)$$

where  $\beta$  are the calculated coefficients;  $n$  is the number of sample points;  $\hat{y}_{(i)}$  is the value of the metamodel's response for the  $i$ -th sample point;  $y_{(i)}$  is the actual value of the response of the computer analysis code in vector  $\mathbf{y}$ .

A more complete discussion on the polynomial metamodels and least squares method can be found in Myers & Montgomery<sup>14</sup>. In the present study full polynomials of 2<sup>nd</sup>, 3<sup>rd</sup>, and 4<sup>th</sup> order have been employed.

##### B. Locally weighted polynomials

Locally weighted polynomial approximation was originally proposed by Cleveland<sup>15</sup>. It was designed to address situations in which the global polynomials do not perform well or cannot be effectively applied without undue effort. The approximation is carried out by pointwise fitting of low-order polynomials to localized subsets of the data. The advantage of this method is that the analyst is not required to specify a global function of the data. However, the method requires considerably higher computational resources.

The assumption of the local polynomial approximation is that near the query point the value of the actual response changes smoothly and can be approximated using a low-order polynomial. The coefficients of the polynomial are then calculated using the weighted least squares method giving the largest weights to the nearest (usually according to the Euclidian distance) sample points and the lowest or zero weights to the farthest sample points.

The coefficients  $\beta$  are calculated by the weighted least squares minimizing

$$\beta = \arg \min_{\beta} \sum_{i=1}^n w(x_{query}, x_{(i)}) (\hat{y}_{(i)} - y_{(i)})^2 \quad (6)$$

where  $w$  is a weight function;  $x_{query}$  is the query point nearest neighbors of which will get the highest weights;  $x_{(i)}$  is the  $i$ -th point in vector  $\mathbf{x}$ . The weight function  $w$  depends on the Euclidean distance (in scaled  $[-1,1]^d$  space) between the point of interest  $x_{query}$  and the points of observations  $x$ . One of the most widely used weight functions is the Gaussian weight function<sup>11</sup>:

$$w(x_{query}, x_{(i)}) = \exp(-\alpha\mu^2) \quad (7)$$

where  $\alpha$  is a coefficient and the  $\mu$  can be calculated as

$$\mu = \frac{\|x_{query} - x_{(i)}\|}{\|x_{query} - x_{farthest}\|} \quad (8)$$

where  $\|\cdot\|$  is the Euclidian norm;  $x_{farthest}$  is the farthest point in the neighborhood of the point  $x_{query}$ . In general, the Gaussian weight function with constant value  $\alpha = 1/2$  is used in local approximations varying only the value of the considered nearest neighbors (unlike in equation (6) where all the sample points are used)<sup>11</sup>. However, in the present study all the sample points have been used and the locality of the approximation has been controlled by varying the value of the coefficient  $\alpha$ . If  $\alpha$  is equal to zero then local approximation transforms into global approximation. The best value of  $\alpha$  is found using the leave-one-out cross-validation technique<sup>16</sup>. In the present study locally weighted polynomials of 2<sup>nd</sup>, 3<sup>rd</sup>, and 4<sup>th</sup> order have been employed.

### C. Multivariate Adaptive Regression Splines

Multivariate Adaptive Regression Splines<sup>17,18</sup> was proposed as a method for flexible regression modeling of high dimensional data (i.e., a large number of input variables). The model takes the form of an expansion in product spline basis functions, where the number of basis functions as well as the parameters associated with each one (product degree and knot locations) are automatically determined by the data through a forward/backward iterative approach. Compared to polynomial approximations, the use of MARS for engineering design is relatively new. However, its application is drawing an increasing attention of the researchers (e.g., Jin et al.<sup>1</sup>).

MARS model can be defined as a sum of basis functions<sup>17,18</sup>:

$$\hat{y} = \beta_0 + \sum_{i=1}^k \beta_i f_i(x) \quad (9)$$

where  $f_i(x)$  is a basis function;  $k$  is the number of basis functions in the model except for the constant basis function  $f_0(x) = 1$  coefficient of which is the  $\beta_0$ . The basis functions are of the form

$$f_i(x) = \prod_{j=1}^{d_i} [s_{ji} (x_{v(j,i)} - t_{ji})]_+ \quad (10)$$

where  $d_i$  is the number of variables (interaction order) in the  $i$ -th basis function;  $s_{ji} = \pm 1$ ;  $x_{v(j,i)}$  is the  $v$ -th variable,  $1 \leq v(j,i) \leq d$ ;  $t_{ji}$  is knot location on each of the corresponding variables. The subscript '+' means that the function is a truncated power function<sup>17,18</sup>. The coefficients  $\beta$  are again determined by the ordinary least squares method (equation (5)). In the present study piecewise-cubic MARS version 3.6 without a specific restriction of the number of basis functions or interaction orders has been employed.

### D. Partial polynomials

Low-order global polynomial approximations have been well accepted in engineering practice, as they require low number of sample points and are computationally very efficient. On other hand they can not approximate highly

nonlinear behavior. Instead, higher-order polynomials can be employed. However, if no special care is taken they tend to overfit the data and produce high errors in regions where the sample points are relatively sparse. One possible remedy for the overfitting problem is employment of the subset selection (also called model building) techniques<sup>13,14</sup>. The techniques are aimed to identify the best subset of polynomial terms (or basis functions) to include in the model and to remove the unnecessary ones, in this manner creating a partial polynomial model (in lieu of ‘full’ model) of increased predictive performance. However, the approach of subset selection assumes that the chosen fixed full set of predefined basis functions (usually just by choosing a fixed maximal order of the polynomial) contains a subset that is sufficient to describe the target relation sufficiently well. Hence the effectiveness of subset selection largely depends on whether or not the predefined set of basis functions contains such a subset.

There exists a different approach for polynomial model building which does not assume a predefined set of basis functions – Adaptive Basis Function Construction<sup>19,20</sup>. The approach allows generating polynomials of arbitrary complexity and order without the requirement to predefine any basis functions or to set the maximal order of the polynomial (or any other hyperparameters) – all the required basis functions are constructed adaptively.

Generally a polynomial model can be defined as a linear summation of basis functions:

$$\hat{y} = \sum_{i=1}^k \beta_i f_i(x) \quad (11)$$

where the coefficients  $\beta$  are still calculated by the ordinary least squares method (equation (5));  $f_i(x)$  is a basis function which generally can be defined as a product of the input variables each raised to some order:

$$f_i(x) = \prod_{j=1}^d x_j^{r_{ij}} \quad (12)$$

where  $r_{ij}$  is the order of the  $j$ -th variable in the  $i$ -th basis function (a non-negative integer). It should be noted that when all  $r_j$ 's of a basis function become equal to 0, the basis function becomes equal to 1, ergo it is the intercept term.

The matrix  $r$  completely defines all the basis functions in the model – each row corresponds to one basis function with all of its orders. Construction of the model is carried out in an iterative manner directly with the matrix  $r$  using five simple so-called model refinement operators which allow adding, copying, modifying, and deleting the rows of  $r$ , i.e., adding, copying, modifying, and deleting the basis functions of the model<sup>19,20</sup>. As a search procedure a modification of the Sequential Floating Forward Selection<sup>21</sup> algorithm is employed while models are evaluated using the Corrected Akaike's Information Criterion<sup>22</sup>. Additionally, in order to lower the model building issues of selection bias and selection instability a technique of model averaging (also called ensembling or combining) is carried out<sup>20</sup>.

### E. Metamodel evaluation

To evaluate the metamodels, 5-fold cross-validation technique<sup>16</sup> has been used where the full data set is divided in five equally-sized subsets. In each of the five cross-validation iterations four of the subsets are used for metamodel building and one subset is used as an independent test set for evaluation of the metamodel. As the metamodel accuracy measure the Relative Root Mean Square Error has been used:

$$RRMSE = 100\% \frac{\sqrt{\frac{1}{n_t} \sum_{i=1}^{n_t} (y_{(i)} - \hat{y}_{(i)})^2}}{STD} \quad (13)$$

where  $n_t$  is the number of test points; STD is the standard deviation in test sample:

$$STD = \sqrt{\frac{1}{n_t} \sum_{i=1}^{n_t} (y_{(i)} - \bar{y})^2} \quad (14)$$

where  $\bar{y}$  is the mean value of all  $y$  values in the test sample. It should be noted that RRMSE and STD are calculated using strictly only the test sample and averaged over all the cross-validation runs.

## V. Metamodeling results

As structural response parameters the following entities have been taken: global deflection of the deck panel  $u$ , the relative deflection ratio between the deck length and the plate deflection  $\Delta u$ , the maximum equivalent stress at the upper plate and the stiffeners  $\sigma_{top}$ ,  $\sigma_{stif}$  and the maximum shear stresses in stiffeners  $\tau$ . A total of 500 sampling points have been generated and a cross-validation procedure together with the RRMSE measure has been carried out comparing the different metamodeling techniques. Two loading scenarios have been selected for metamodeling: the concentrated three-point bending load case and uniformly distributed load case with simple support boundary conditions. The obtained results are summarized in Tables 3 and 4.

**Table 3. Cross-validation RRMSE errors of different metamodels built using 500 sample points in the case of concentrated three-point bending load**

Metamodels	Parametric polynomial approximations				Locally weighted polynomials			MARS
	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>	ABFC	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>	
Panel deflection, $u$	27.89	13.04	6.71	0.81	16.75	9.90	6.12	3.00
Comparative deflection, $\Delta u$	14.09	3.95	1.57	0.45	7.09	2.74	1.47	2.03
Max_stresses, $\sigma_{top}$	14.36	4.74	2.32	1.33	7.35	3.69	2.29	2.03
Max_stresses, $\sigma_{stif}$	12.56	5.45	3.93	3.00	7.28	4.55	4.03	3.92
Max_shear_stresses, $\tau$	10.32	7.42	7.73	2.18	7.91	7.11	7.71	5.82

**Table 4. Cross-validation RRMSE errors of different metamodels built using 500 sample points in the case of uniformly distributed load**

Metamodels	Parametric polynomial approximations				Locally weighted polynomials			MARS
	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>	ABFC	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>	
Panel deflection, $u$	34.23	18.04	11.54	1.07	23.15	14.15	10.47	5.26
Comparative deflection, $\Delta u$	45.26	32.81	29.44	13.27	31.83	27.83	27.88	19.58
Max_stresses, $\sigma_{top}$	16.94	6.36	5.59	3.56	7.88	5.28	5.52	4.94
Max_stresses, $\sigma_{stif}$	16.18	7.61	4.59	1.67	10.01	5.97	3.84	3.54
Max_shear_stresses, $\tau$	10.36	8.09	11.38	7.32	8.30	8.05	11.56	11.28

The conventional 2<sup>nd</sup> order polynomials, which are mostly associated with engineering problems of the response surface methodology, gave the worst approximation results for almost all the response values. It has been noted that locally weighted polynomials of the 2<sup>nd</sup> order considerably increased the predictive performance. As overall observation could be stated that, by increasing the order of polynomials, the approximation performance rose, however the higher was the order the smaller was the improvement of the locally weighted polynomials over the global ones. Additionally, it should be expected that decreasing the number of the sampling points would lead the full polynomials of higher orders to overfitting the data thus rapidly reducing their predictive performance.

The best results were obtained using the ABFC approach, leaving the MARS technique as the second best. One can conclude that, although using higher order global polynomials or locally weighted polynomials can improve the predictive performance, an elaborated adaptive search for partial polynomials or regression splines has capabilities to provide an even further performance boost.

Additionally, it has been investigated how rapidly the performance of the metamodeling methods would deteriorate while decreasing the number of sample points. For the case of concentrated three-point bending load additional results for 200 sample points instead of 500 were obtained. The results are summarized in Table 5.

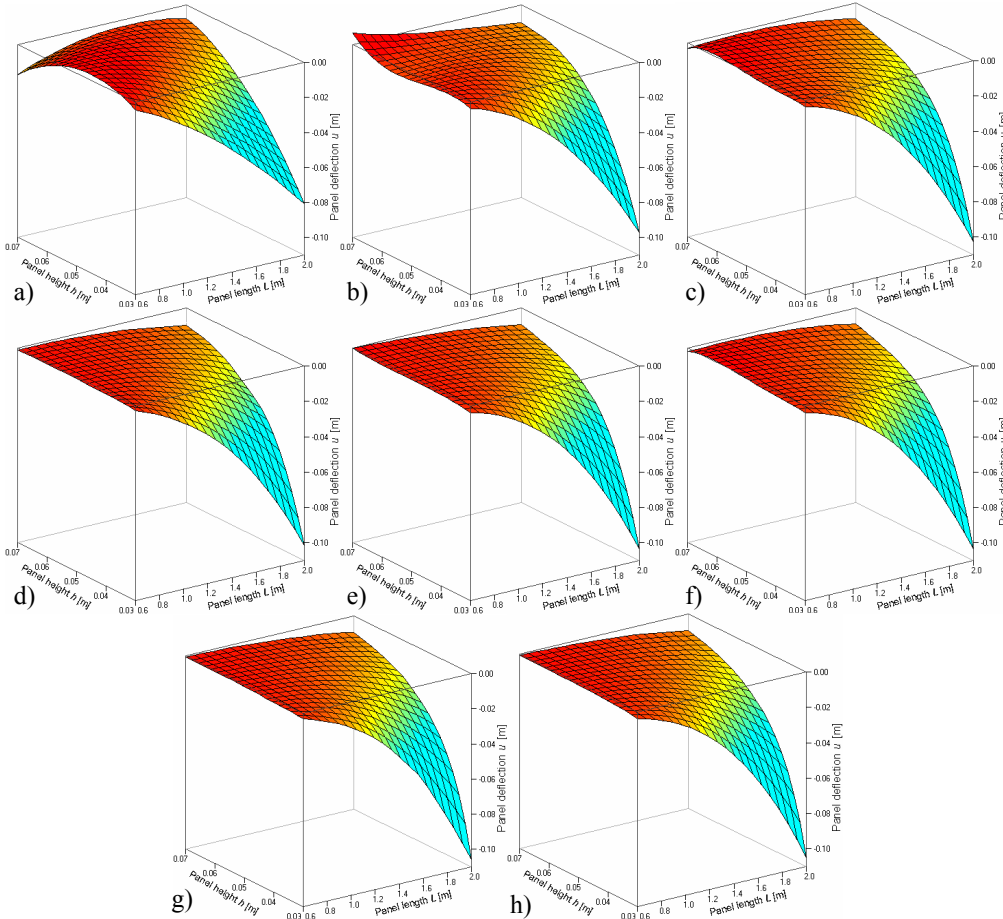
With the reduced number of sample points the 4<sup>th</sup> order polynomials became impossible to use because the number of their basis functions was now higher than the number of the sample points. For all metamodels a reduction in their predictive performance has been observed, however the most rapid performance deterioration was for MARS metamodels. The best results were still obtained using the ABFC approach. Apparently, compared to the other techniques, partial polynomials are relatively more efficient than others when a small dataset of sample points is used while reducing the number of basis functions in a polynomial model.



**Table 5. Cross-validation RRMSE errors of different metamodells built using 200 sample points in the case of concentrated three-point bending load**

Metamodels	Parametric polynomial approximations				Locally weighted polynomials			MARS
	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>	ABFC	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>	
Panel deflection, $u$	25.82	14.51	N/A	1.87	16.92	13.68	N/A	7.51
Comparative deflection, $\Delta u$	16.48	5.90	N/A	1.38	11.50	5.22	N/A	4.69
Max_stresses, $\sigma_{top}$	14.07	5.73	N/A	1.79	9.08	5.55	N/A	3.64
Max_stresses, $\sigma_{stif}$	12.18	6.03	N/A	4.04	8.80	6.06	N/A	6.91
Max_shear_stresses, $\tau$	11.60	10.56	N/A	7.35	10.81	10.68	N/A	8.39

Moreover, three-dimensional graphical validations of the developed metamodells for the deck panel deflection  $u$  versus the panel length  $L$  and the panel height  $h$  parameters in the case of concentrated three-point bending load with 500 sample points were carried out as presented in Figure 5. By graphical validation one can easily identify that low order global polynomial approximations at the maximum height and the minimum length behave differently than expected. In particular the second order and to a lesser extent also the fourth order polynomial surface plots show a decrease of stiffness when increasing the panel height. On the other hand, the third order polynomial function shows non-negative deflection at the boundaries, which would indicate bending against gravity. The locally weighted polynomials reduce the unwanted behavior. However, by zooming in at the region of the maximum height and the minimum length, the surface plots reveal that the higher is the order of the locally weighted polynomials the more they behave like global polynomials resulting again in the unwanted behavior.



**Figure 5. Graphical validation of surrogate models for panel deflection in the case of concentrated load.** Full global polynomials of 2<sup>nd</sup> (a), 3<sup>rd</sup> (b), and 4<sup>th</sup> (c) order; locally weighted polynomials of 2<sup>nd</sup> (d), 3<sup>rd</sup> (e), and 4<sup>th</sup> (f) order; MARS (g); ABFC (h)

Overall, the low-order locally weighted polynomial approximation, MARS, and the ABFC gave the best overall perspective of structural behavior by creating a plateau-like surface for the height of stiffened panel designs. It seems that here for an optimization procedure all the three methods can be used with some confidence however for what-if analysis the ABFC would be the most accurate.

## VI. Conclusion

The comparison study between parametric and non-parametric metamodels has been elaborated for design of pultruded GFRP deck structures under the bending load. Two loading scenarios have been selected to investigate the metamodeling efficiency and have been validated with physical experiments. It has been concluded that the partial polynomials and MARS are capable to improve the prediction accuracy compared to conventional 2<sup>nd</sup> order polynomials, which frequently are associated with engineering problems of the response surface methodology. In particular, the bending deflection responses could be improved by an order of magnitude compared to the 2<sup>nd</sup> order polynomials. In contrary, the improvement in approximation prediction for equivalent stresses and shear stresses are less efficient. Elaborated metamodels have the capability to be used in implementation of optimum design methodology for the bended deck structures.

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